

PROBABILITY

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Win or Lose?

Success - s Failure - f

$$\text{Odds: } \frac{s}{f}$$

$$\text{Probability: } \frac{s}{s+f}$$

The odds of rolling a 6 with a six-sided die are 1 to 5 – one way to get a 6, and 5 ways not to.

The probability of rolling a 6 with a six-sided die is 1 in 6 – one way to get a 6 out of 6 possibilities.

Terminology

sample space: every possible outcome.

equiprobable space: all outcomes are equally likely; keywords – “at random.”

success: the desired outcome.

failure: any undesired outcome.

events -

impossible: no success is possible.

simple: exactly one success is possible.

compound: more than one success is possible.

certain: only success is possible.

Probability of an Event

The probability of an event A happening is written as:

$$P(A)$$

Dependent Probabilities

multiplying probabilities

If a first event has no bearing on the outcome of a second event then they are **independent events**. The probability of both of those events happening is given by:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

If a first event affects the outcome of a second event, then they are **dependent events**. The probability of both of those events happening is given by:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

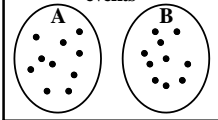
where $P(B|A)$ is “the probability of B given A” (the denominator $[s + f]$ gets smaller here).

Hint: A key phrase to look for is “with/without replacement.” “With replacement” the events are independent; “Without replacement” the events are dependent.

Inclusive Probabilities

adding probabilities

Mutually exclusive events

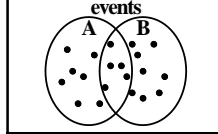


If two events are not in the same set of possibilities (same sample space), then they are **mutually exclusive**. The probability of one or the other happening is given by:

$$P(A \text{ or } B) = P(A) + P(B)$$

If two events have overlapping sets of possibilities then they are **inclusive**. The probability of one or the other happening is given by:

Inclusive events



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

“n factorial” means...

$$n! = 1 \times 2 \times 3 \times \dots \times n$$

Factorials!

memorize!

$$\begin{array}{lll} 0! = 1 & 2! = 2 & 4! = 24 \\ 1! = 1 & 3! = 6 & 5! = 120 \end{array}$$

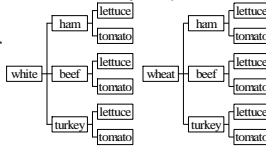
Counting

Note: With these kinds of problems the first step is to **always** figure out which kind of problem it is. Practice that skill, get good at that and you'll have no problem applying the formulas. There are four general types of problems you should be familiar with.

The Sandwich problem

The fundamental counting principle: If some number of events can happen m, n, p, \dots different ways, then the number of ways all those events can happen is $m \cdot n \cdot p \dots$

Given 2 kinds of bread, 3 kinds of meat, and 2 kinds of veggies, how many different sandwiches can be made with one of each? $2 \cdot 3 \cdot 2 = 12$



The License Plate problem

A license plate consists of a sequence of numbers and letters. How many plates are available in that sequence if **repeating** is **allowed**?

$$\begin{array}{l} \dots \text{can repeat} \\ \text{059 AXZ} \\ 10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \\ = 17,576,000 \text{ plates} \end{array}$$

$$\begin{array}{l} \dots \text{can not repeat} \\ \text{059 AXZ} \\ 10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24 \\ = 11,232,000 \text{ plates} \end{array}$$

If **repeating is not allowed**, how many plates are available in the sequence?

Permutations

The Train: A number n of boxcars are being arranged in a row, how many different arrangements

are possible? If $n = 5, \dots$

car 1 n places	car 2 4 places	car 3 3 places	car 4 2 places	car n 1 place
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$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

The Race: If we only care about the first r places, like in a race,

the rest of the places don't matter. How many ways can the first three places be won in this race? 60

$$nPr = \frac{n!}{(n-r)!}$$

The “permutation” CBA is different than ABC – the order matters!

Combinations

The Poker hand: A combination is any group of r objects chosen from a group of n objects in any order. How many combinations of 5 cards can be dealt from a deck with 52 cards? **2,598,960**

$$nC_r = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

(Notice this is the same formula as for binomial coefficients above.)

Multiply or add? When combining combinations, if A **and** B must happen then **multiply** combinations; if only A **or** B can happen then **add** combinations.