PHYSICS PHOR PHUN
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BINOMIAL THEOREM AND COUNTING

MATH AND SCIENCE TUTORING
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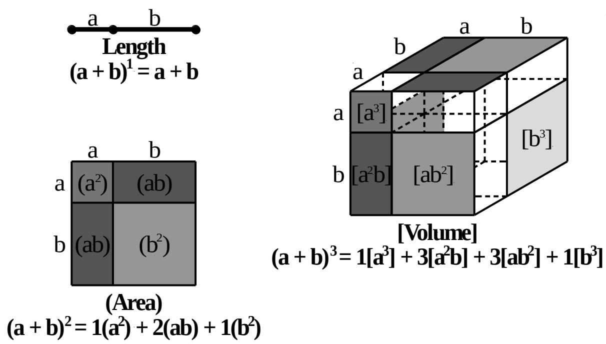
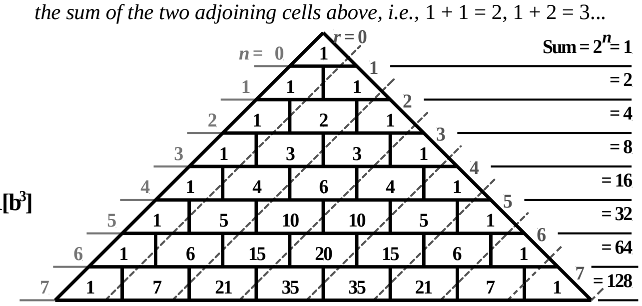
BINOMIAL THEOREM

The binomial (a + b), its perfect square $(a + b)^2 = a^2 + 2ab + b^2$, and its perfect cube $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$, have a real meaning shown in the following pictures on the left. We can't draw a picture for higher powers like $(a + b)^4$ but we can still do the math. One method is Pascal's triangle on the right.



Pascal's Triangle**:** *Divide a triangle into rows. Split the second row with a vertical line from the center of the cell above; divide the remaining rows the same way.* **1***'s go in all the edge cells. All other cells get the sum of the two adjoining cells above, i.e., 1 + 1 = 2, 1 + 2 = 3...*



Binomial expansion**:** *To avoid multiplying* $(\boldsymbol{a} + \boldsymbol{b})^n$ *just write '****ab****'* $n + 1$ *times. Number the exponents on* ***a****, left to right, from* ***n*** *to* ***zero****; the* ***b*** *exponents from* ***zero*** *to* ***n****. Coefficients for each '****ab****' term are the cells from row* ***n*** *of the triangle; first term gets* $r = 0$*, last term gets* $r = n$*. Signs are all +'ve for (a + b), and alternate +, —, +, —... for (a – b) always starting +'ve. Substitute values or expressions for* ***a*** *and* ***b****.*

$$(a\pm b)^7 = +\mathbf{1}a^7b^0 \pm \mathbf{7}a^6b^1 + \mathbf{21}a^5b^2 \pm \mathbf{35}a^4b^3 + \mathbf{35}a^3b^4 \pm \mathbf{21}a^2b^5 + \mathbf{7}a^1b^6 \pm \mathbf{1}a^0b^7$$

Calculating Binomial Coefficients

Beyond row 6 or 7, Pascal's triangle is not the best tool for this job. Your calculator should have a "choose" function: n**C**r*. The n is the n from* $(a + b)^n$ *(blue numbers on triangle); the r (red numbers on triangle) is which term, zero through n, to get the coefficient for.*

Formula*

$$n\mathbf{C}r = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

****The ! means "factorial" as discussed below.*

Pascal's Properties

*Every binomial coefficient can be found by adding cells, by the choose function, n***C***r, or by the formula for the choose function.*

The cell values are symmetric about the vertical centerline.

The cell values of each row n add up to 2^n*;* 2^n *is the number of subsets any set of n objects contains.*

On a large scale (100's of rows) many regular patterns of groupings with common factors appear – 5's are very strong.

Factorials!

"n factorial" means...

$$n! = 1\times 2\times 3\times\cdots\times n$$

memorize!

0! = 1	2! = 2	4! = 24
1! = 1	3! = 6	5! = 120

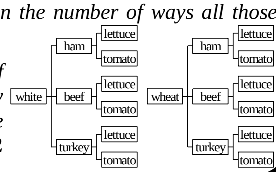
Counting

Note**:** *With these kinds of problems the first step is to* ***always*** *figure out which kind of problem it is. Practice that skill, get good at that and you'll have no problem applying the formulas. There are four general types of problems you should be familiar with.*

The Sandwich problem

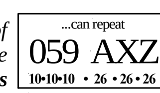
The fundamental counting principle**:** *If some number of events can happen m, n, p... different ways, then the number of ways all those events can happen is m•n•p•...*

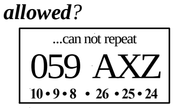
Given 2 kinds of bread, 3 kinds of meat, and 2 kinds of veggies, how many different sandwiches can be made with one of each? **2•3•2 = 12**



The License Plate problem

A license plate consists of a sequence of numbers and letters. How many plates are available in that sequence if ***repeating is allowed****?*

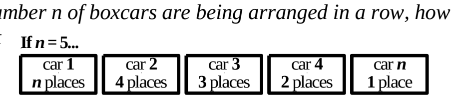




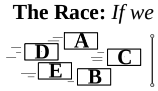
If ***repeating is not allowed****, how many plates are available in the sequence?*

Permutations

The Train: *A number n of boxcars are being arranged in a row, how many different arrangements are possible?* ***n*****!**



The Race: *If we only care about the first r places, like in a race, the rest of the places don't matter. How many ways can the first three places be won in this race?* **60**



$$n\mathbf{P}r = \frac{n!}{(n-r)!}$$

The "permutation" CBA is different than ABC – the ***order matters****!*

Combinations

The Poker hand: *A combination is any group of r objects chosen from a group of n objects in any order. How many combinations of 5 cards can be dealt from a deck with 52 cards?* **2,598,960**

$$n\mathbf{C}r = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

(*Notice this is the same formula as for binomial coefficients above.)*
Multiply or add? *When combining combinations, if A* **and** *B must happen then* ***multiply*** *combinations; if only A* **or** *B can happen then* ***add*** *combinations.*